Estimate :- An estimate is a statement made to find an unknown Population Parameter.

Estimator :- The Procedure or rule to determine an unknown Population Parameter is called an Estimator.

Estimators are two types.

(a) Point estimation

(b) Interval estimation.

Statistical Estimation :- It is a Part of statistical inference where a Population Parameter is estimated from the Corresponding Sample statistics.

Point Estimation :- A Point estimation of a Parameter is a statistical estimation where the Parameter is estimated by a Single numerical value from sample data.

Definition :- A Point estimate of a Parameter θ is a Single numerical value, which is computed from a given Sample and serves as an approximation of the unknown Exact value of the Parameter.

Definition :- A Point estimator is a statistic for estimating the Population Parameter θ and will be denoted by $\hat{\theta}$.

Properties of Estimation :- An estimator is not Expected to estimate the Population Parameter without error. An estimator should be close to the true value of unknown Parameter.

Unbiased Estimator :- Let $\hat{\theta}$ be an estimator of θ. The statistic θ is said to be an unbiased estimator, or its value an unbiased estimate, if and only if the mean or Expected value of $\hat{\theta}$ is Equal to θ. This is equivalent to say that the mean of the Probability distribution of $\hat{\theta}$ is Equal to θ. An estimator possessing this Property is said to be unbiased.

A statistic or Point estimator $\hat{\theta}$ is said to be an unbiased estimator of the Parameter θ if $E(\hat{\theta}) = \theta$.

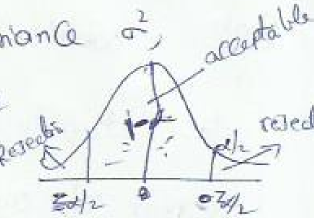
Point estimation : In a sample derived from an unknown Population Parameter, the Calculation of a single value is done as an estimate. The Procedure for determing the Parameter is called Point estimation. The Point estimate of a Population Parameter (θ) is a Single numerical value.

Variance of a Point estimator :- If $\hat{\theta}_1$ and $\hat{\theta}_2$ are two unbaised estimators of the Same Population Parameter θ, we would choose the estimator whose Sampling distribution has the Smaller variance. Hence if $\sigma^2_{\hat{\theta}_1} < \sigma^2_{\hat{\theta}_2}$, we say that $\hat{\theta}_1$ is a more efficient estimator of θ than $\hat{\theta}_2$.

Most Efficient Estimator :- If we consider all possible unbaised estimators of some Parameter θ, the one with the Smallest variance is called the Most efficient estimator of θ.

Good Estimator :- An estimator is said to be a good estimator if it is (i) unbaised (ii) Consistent (iii) Efficient.

Confidence interval :- If $\bar{x}$ is the mean of a random sample of size n from the population with known variance σ^2, $(1-\alpha)100\%$ confidence interval for μ is given by



$$\bar{x} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{x} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

where $z_{\alpha/2}$ is the z-value leaving an area of $\alpha/2$ to the right

So, the maximum error of estimate E with $(1-\alpha)$ probability is given by

(Maximum Error) $E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$.

Thus in the Point Estimation of Population Mean μ with Sample Mean $\bar{x}$ for a large random sample $(n \geq 30)$ one can assert with probability $(1-\alpha)$ that the error $|\bar{x}-\mu|$ will not exceed $z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

Sample Size : where α, σ, E are known then the Sample Size n is given by

$$n = \left[\frac{z_{\alpha/2}\,\sigma}{E}\right]^2$$

Note when σ is unknown In this case σ is replaced by S, the S.D of Sample to determine E

Thus the Maximum Error Estimate $E = t_{\alpha/2} \frac{S}{\sqrt{n}}$ with $(1-\alpha)$ Probability.

Maximum Error of Estimate E for large Sample

$$P\left(\bar{x} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = 1-\alpha.$$

Maximum Error of Estimate for Small Samples

$$P\left(\bar{x} - t_{\alpha/2}\left(\frac{S}{\sqrt{n}}\right) < \mu < \bar{x} + t_{\alpha/2}\frac{S}{\sqrt{n}}\right) = 1-\alpha.$$

Confidence interval for μ; σ unknown

Prob. what is the Size of the Smallest Sample required to estimate an unknown Proportion to within a Maximum error of 0.06 with atleast 95% Confidence.

Sol. we are given

Maximum error $E = 0.06$

Confidence limit $= 95\%$

i.e $(1-\alpha)100 = 95$

$1-\alpha = 0.95$

$\alpha = 0.05$

$\alpha/2 = 0.025$

$\therefore z_{\alpha/2} = 1.96$

Here P is not given, So we take $P = \frac{1}{2}$, Then $Q = \frac{1}{2}$.

Hence $n = \left(\frac{z_{\alpha/2}}{E}\right)^2 (PQ)$

when P is unknown sample size $n = \frac{1}{4}\left[\frac{z_{\alpha/2}}{E}\right]^2$.

$$n = \frac{1}{4}\left[\frac{1.96}{0.06}\right]^2$$

$= 266.78$

$\simeq 267.$

Prob. If we can assert with 95% that the Maximum Error is 0.05 and $P = 0.2$ find the Sample size.

Sol. Given $P = 0.2$, $E = 0.05$.

we have $Q = 1-P \Rightarrow 1-0.2 = 0.8$.

$z_{\alpha/2} = 1.96$ (for 95%)

Maximum Error $E = Z_{\alpha/2}\sqrt{\frac{PQ}{n}}$

$$0.05 = 1.96\sqrt{\frac{(0.2)(0.8)}{n}}$$

$$n = \frac{0.2\times 0.8\times(1.96)^2}{(0.05)^2} = 246$$

Prob. Assuming that $\sigma = 20.0$ how large a random Sample be taken to assert with Probability 0.95 that the Sample Mean will not differ from the true mean by more than 3.0

Sol. Given Maximum Error $E = 3.0$ & $\sigma = 20.0$

$$Z_{\alpha/2} = 1.96$$

We know that $n = \left(\frac{Z_{\alpha/2}\cdot\sigma}{E}\right)^2$

$$= \left(\frac{1.96\times 20}{3}\right)^2$$

$$n \simeq 171$$

Prob. It is desired to estimate the Mean number of hours of Continuous use until a Certain Computer will first require repairs. If it can be assumed that $\sigma = 48$ hours., how large a Sample be needed so that one will be able to assert with 90% Confidence that the Sample mean is off by at most 10 hours.

Sol: Maximum Error $E = 10$ hrs.

$\sigma = 48$ hours

$Z_{\alpha/2} = 1.645$ (for 90%)

$$n = \left(\frac{Z_{\alpha/2}\,\sigma}{E}\right)^2 = \left(\frac{1.645 \times 48}{10}\right)^2$$

$$= 62.3$$
$$\simeq 62.$$

Hence Sample Size = 62.

Prob. What is the Maximum Error one can Expect to make with Probability 0.90 when using the mean of a random Sample of Size $n = 64$ to estimate the Mean of Population with $\sigma^2 = 2.56$

Sol: $n = 64$, the Probability $= 0.90$

$\sigma^2 = 2.56 \Rightarrow \sigma = \sqrt{2.56} = 1.6$

Confidence limit $= 90\%$

$1 - \alpha = \frac{90}{100}$

$1 - \alpha = 0.9.$

$\alpha = 0.1$

$\alpha/2 = 0.05$

$Z_{\alpha/2} = 1.645$

Hence Maximum Error $E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 1.645 \times \frac{16}{\sqrt{64}}$

$= 0.329$

Prob. A random Sample of size 81 was taken whose variance is 20.25 and Mean is 32, Construct 98% Confidence interval.

Sol. Given $\bar{x}$ = Sample Mean. = 32.

$n = 81$

$\sigma^2 = 20.25$

$\sigma = \sqrt{20.25} = 4.5$

$z_{\alpha/2} = 2.33$ (98%)

$1-\alpha = \frac{98}{100}$ $\quad \begin{array}{r} 1.00 \\ 0.98 \\ \hline 0.02 \end{array}$

$1-\alpha = 0.98$

$\alpha = 0.02$

$\alpha/2 = 0.01$

we know that 98% Confidence interval is

$$\left[\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$

$$z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 2.33\left(\frac{4.5}{\sqrt{81}}\right) = 1.165$$

Confidence interval $(32 - 1.165,\ 32 + 1.165)$

$(30.835,\ 33.165)$

Prob The mean of random sample is an unbiased estimate of the mean of the population 3, 6, 9, 15, 27

(i) list of all possible samples of size 3 that can be taken without replacement from the finite population.

(ii) Calculate the mean of each of the samples listed in (i) and assigning each sample a probability of 1/10. which is verify that the mean of these $\bar{x}$ is equal to 12. which is equal to the mean of the population θ. i.e $E(\bar{x}) = \theta$. i.e P.T $\bar{x}$ is an unbaised estimate of θ.

Sol. (i) The possible samples of size 3 taken from 3, 6, 9, 15, 27 without replacement, are $5C_3 = 10$ samples

ie (3, 6, 9) (3, 6, 15) (3, 6, 27) (6, 9, 15) (6, 9, 27) (3, 9, 15) (3, 9, 27) (9, 15, 27) (6, 15, 27), (3, 15, 27)

(ii) mean of the population $\theta = \frac{3+6+9+15+27}{5} = 12$

mean of the samples are 6, 8, 12, 10, 14, 9, 13, 17, 16, 55

Probability assigned to Each one is $\frac{1}{10}$ Each

$\bar{x}$	6	8	12	10	14	9	13	17	16	15
$P(\bar{x})$	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10

$$E(\bar{x}) = 6\left(\frac{1}{10}\right) + 8\left(\frac{1}{10}\right) + \frac{12}{10} + \frac{10}{10} + \frac{14}{10} + \frac{9}{10} + \frac{13}{10} + \frac{17}{10} + \frac{16}{10} + \frac{15}{10}$$

$$= 12 = \theta$$

$\therefore E(\bar{x}) = \theta$

$\bar{x}$ is a unbaised estimate of θ

ie the Mean of a random sample is an unbaised estimator of the Mean of the Population.

Bayesian Estimation :- Bayesian estimation is used to obtain Mean and variance of poserior distribution of a Population. If the Prior distribution Parameters mean μ_0 and variance σ_0^2 of a population are known and when they are Combined with the direct Sample statistics, say mean $\bar{x}$, then it is Possible to estimate the Posterior distribution Parameters of a given Population. This is called Bayesian estimation.

Let μ_0 be the Prior Mean

σ_0^2 be the Prior variance

$\bar{x}$ be the Sample mean

n be the Sample Size and

σ^2 be the Sample variance.

then the Posterior distribution Parameters can well be approximated by normal distribution.

The Mean of Posterior distribution $\mu_1 = \dfrac{n\bar{x}\sigma^2 + \mu_0\sigma_0^2}{n\sigma^2 + \sigma_0^2}$

The variance of Posterior distribution, $\sigma_1^2 = \dfrac{\sigma^2\sigma_0^2}{n\sigma^2 + \sigma_0^2}$

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Prob. There are 25 Cards, on four of them written 5, on six of them 10, on five of them 15, on three of them 20, on four of them 25 and on three of them 30.

One each of 5, 10, 15, 20, 25, 30 are selected without replacement. find

(a) the mean of the Population

(b) S.D of the Population

(c) Means of Means of Sampling distribution.

d) S.D of means of Sampling distribution.

(a) Mean of the Population

$$\mu = \frac{(4\times5)+(6\times10)+(5\times15)+(3\times20)+(4\times25)+(3\times30)}{25}$$

$$= \frac{405}{25} = 16.2$$

(b) S.D of the Population $= \sqrt{\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{N}}$

$$= \sqrt{\frac{4(5-16.2)^2+6(10-16.2)^2+5(15-16.2)^2+3(20-16.2)^2+4(25-16.2)^2+3(30-16.2)^2}{25}}$$

$$= \sqrt{\frac{1664}{25}} = 8.16$$

(c) Population is $\{(5,5,5,5)\,(10,10,10,10,10)\,(15,15,15,15,15)\,(20,20,20)\,(25,25,25,25)\,(30,30,30)\}$

Samples are selecting at $5 = \frac{4}{25}$

Probability of selecting a $10 = 6/25$

" " " a $15 = 5/25$

" " " " $20 = 3/25$

" " " " $25 = 4/25$

" " " " $30 = 3/25$

x	5	10	15	20	25	30
$P(x)$	4/25	6/25	5/25	3/25	4/25	3/25

$$\mu_{\bar{x}} = \Sigma x P(x) = \left(5\times\frac{4}{25}\right) + \left(10\times\frac{6}{25}\right) + \left(15\times\frac{5}{25}\right) + \left(20\times\frac{3}{25}\right) + \left(25\times\frac{4}{25}\right) + \left(30\times\frac{3}{25}\right)$$

$$= 16.2$$

(d) S.D of Means of Sampling distribution $\sigma_{\bar{x}} = \sqrt{\Sigma (x-\mu_{\bar{x}})^2 p(x)}$

$$= \sqrt{(5-16.2)^2\frac{4}{25} + (10-16.2)^2\frac{6}{25} + (15-16.2)^2\frac{5}{25} + (20-16.2)^2\frac{3}{25} + (25-16.2)^2\left(\frac{4}{25}\right) + (30-16.2)^2\frac{3}{25}}$$

$$= \sqrt{66.56}$$
$$= 8.16.$$

Prob. Take 30 slips of Paper and label 5 Each −4 and 4, four Each −3 and 3, three Each −2 and 2, 2 Each −1, 0 and 1. If Each slip of the Paper has the Same Probability of being drawn, find the Probabilities of getting −4, −3, −2, −1, 0, 1, 2, 3, 4. and find the Mean and variance of this distribution of Means.

Sol. Probabilities of getting −4, −3, −2, −1, 0, 1, 2, 3, & 4 are

x	−4	−3	−2	−1	0	1	2	3	4
$P(x)$	1/6	2/15	1/10	1/15	1/15	1/15	1/10	2/15	1/6

Means of Sampling distribution.

$$\frac{\left(-4\times\frac{1}{6}\right) + \left(-3\times\frac{2}{15}\right) + \left(-2\times\frac{1}{10}\right) + \left(-1\times\frac{1}{15}\right) + \left(1\times\frac{1}{15}\right) + \left(2\times\frac{1}{10}\right) + \left(3\times\frac{2}{15}\right) + \left(4\times\frac{1}{6}\right)}{9}$$

$= 0$ Mean of the Sampling distribution of Means, $\mu_{\bar{x}} = 0$

Mean of the Sampling distribution of Means

Variance of the Sampling distribution of Means

$$\sigma_{\bar{x}}^2 = \Sigma (x-\mu_{\bar{x}})^2 p(x)$$
$$= (-4)^2\left(\frac{1}{6}\right) + (-3)^2\left(\frac{2}{15}\right) + (-2)^2\left(\frac{1}{10}\right) + (-1)^2\left(\frac{1}{15}\right) + (1)^2\left(\frac{1}{15}\right) + 2^2\left(\frac{1}{10}\right) + 3^2\left(\frac{2}{15}\right) + 4^2\left(\frac{1}{6}\right)$$

$$= \cancel{80+36+12}\ \frac{16}{6} + \frac{18}{15} + \frac{4}{10} + \frac{1}{15} + \frac{1}{15} + \frac{4}{10} + \frac{18}{15} + \frac{16}{6}$$

$$= \frac{80+36+12+2+2+12+36+80}{30} = \frac{260}{30} = 8.67$$