Unit-II

Theory of Inference for statement Calculus.

The main aim of logic is to provide rules of inference to infer a conclusion from certain premises. The theory associated with rules of inference is known as inference theory. Premises means set of assumptions, axioms and hypothesis.

If a conclusion is derived from a set of premises by using accepted rules of reasoning, then such a process of derivation is called a deduction or a formal proof and the argument is called a valid argument or conclusion is called a valid conclusion.

Note:- Premises means set of assumptions, axioms and hypothesis.

Let A, B be two statement formulas, we say that "B logically follows from A" (or) "B is valid conclusion (consequence) of the premise A" iff $A \rightarrow B$ is a tautology i.e $A \rightarrow B$.

To determine whether the conclusion logically follows from the given premises, we use the following methods:

1) Truth table method (2) without using truth table method.

Problems.

1. Determine whether the conclusion 'c' follows logically from the hypothesis H_1 & H_2.

(a) $H_1 : P \to Q$. $C : P \to (P \wedge Q)$

As $\sim P \vee (P \wedge Q)$
$(\sim P \vee P) \wedge (\sim P \vee Q)$
$T \wedge$

P	Q	$P \to Q : H_1$	$P \wedge Q$	$P \to (P \wedge Q) : C$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

'c' follows from the premise H_1, because 1^{st}, 3rd, 4th rows H_1 value is T & c value is T.

The conclusion is valid.

(b) $H_1 : \sim P$, $H_2 = P \vee Q$. $C : Q$.

P	~P	Q	P∨Q
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F.

P	Q	~P	P∨Q	C: Q
T	T	F	T	
T	F	F	T	
F	T	T	T	
F	F	T	F	

$\sim P \wedge P \vee Q$.
T

The conclusion is valid, c follows from the Premises H_1 & H_2

(iii) $H_1 : P \rightarrow Q$ $H_2 : \sim P$; $C = P$

P	Q	~P	$P \rightarrow Q = (\sim P \vee Q)$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

The conclusion is not valid.
'c' does not follow H_1, H_2
in 4th row H_1, H_2 is T but c value is F

2 (i) $H_1: P \to Q$ $\quad H_2: P$ $\quad C: Q$

(ii) $H_1: P \to Q$ $\quad H_2: \sim P$ $\quad C: Q$

(iii) $H_1: P \to Q$ $\quad H_2: \sim(P \wedge Q)$ $\quad C: \sim P$

(iv) $H_1: \sim P$ $\quad H_2: P \Leftrightarrow Q$, $\quad C: \sim(P \wedge Q)$

(v) $H: P \to Q$ $\quad H_2: Q$ $\quad C: P$.

P	Q	$\sim P$	$P \to Q$ $(\sim P \vee Q)$	$P \wedge Q$	$\sim(P \wedge Q)$	$P \Leftrightarrow Q$ $(P \to Q) \wedge (Q \to P)$ $(\sim P \vee Q) \wedge (\sim Q \vee P)$
T	T	F	T	T	F	T
T	F	T	T	F	T	F
F	T	T	T	F	T	F
F	F	F	F	F	T	T

(i) $H_1: P \to Q$ $\quad H_2: P$ $\quad C: Q$

It is valid. we observe that the I^{st} row is the only row in which both the Premises have the Value T. The Conclusion also has the Value T in that row. Hence it is valid.

(ii) $H_1: P \to Q$ $\quad H_2: \sim P$ $\quad C: Q$

It is not valid. we observe that the 3rd, 4th rows. The Conclusion Q is true only in the 3rd row but not in the 4th. Hence the Conclusion is not Valid

(iii) $H_1: P \to Q$ $\quad H_2: \sim(P \wedge Q)$ $\quad C: \sim P$

It is valid. observe the 3rd, 4th rows. The conclusion $\sim P$ is true in these 2 rows. Hence it is valid.

(iv) $H_1: \sim P$ $\quad H_2: P \Leftrightarrow Q$ $\quad C: \sim(P \wedge Q)$

It is Valid. 4th row is only row in which both Premises have the value T. The Conclusion $\sim(P \wedge Q)$ has the Value T in that row.

(v) $H_1: P \to Q$ $\quad H_2: Q$ $\quad C: P$

It is not Valid: observe the I^{st} & 3rd rows. The conclusion P is true only in I^{st} row, but not in 3rd row & hence the Conclusion is not Valid.

Rules of Inferences (without using truth table)

There are two rules of inference which are called rules P & T.

The following are two important rules of inferences:

~~Rule P~~ **Rule P**:- A Premise may be introduced at any Point in deviation.

Rule T: A formula 'S' may be introduced by any one or more of the preceeding formulas in the deviation.

Implication formulas

I_1 : $P \wedge Q \Rightarrow P$
I_2 : $P \wedge Q \Rightarrow Q$ } simplification.

I_3 : $P \Rightarrow P \vee Q$
I_4 : $Q \Rightarrow P \vee Q$ } (addition)

I_5 : $\sim P \Rightarrow P \to Q$

I_6 : $Q \Rightarrow P \to Q$

I_7 : $\sim(P \to Q) \Rightarrow P$

I_8 : $\sim(P \to Q) \Rightarrow \sim Q$

I_9 : $P, Q \Rightarrow P \wedge Q$

I_{10} : $\sim P, P \vee Q \Rightarrow Q$ (disjunctive ~~syllogism~~ syllogism)

I_{11} : $P, P \to Q \Rightarrow Q$ (modus ponens)

I_{12} : $\sim Q, P \to Q \Rightarrow \sim P$ (modus tollens)

I_{13} : $P \to Q, Q \to R \Rightarrow P \to R$ hypothetical (syllogism)

I_{14} : $P \vee Q, P \to Q, Q \to R \Rightarrow R$ deliemma (deliemma)

is not valid.

Equivalent formulae

$E_1 : \sim(\sim P) \Leftrightarrow P$ double negation laws

$E_2 : P \wedge Q \Leftrightarrow Q \wedge P$
$E_3 : P \vee Q \Leftrightarrow Q \vee P$ Commutative laws

$E_4 : P \vee (Q \vee R) \Leftrightarrow (P \vee Q) \vee R$
$E_5 : P \wedge (Q \wedge R) \Leftrightarrow (P \wedge Q) \wedge R$ Associative laws

$E_6 : P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$
$E_7 : P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$ Distributive laws

$E_8 : \sim(P \wedge Q) \Leftrightarrow \sim P \vee \sim Q$
$E_9 : \sim(P \vee Q) \Leftrightarrow \sim P \wedge \sim Q$ Demorgan's laws

$E_{10} : P \vee P \Leftrightarrow P$
$E_{11} : P \wedge P \Leftrightarrow P$ idempotent law.

$E_{12} : R \vee (P \wedge \sim P) \Leftrightarrow R$

$E_{13} : R \wedge (P \vee \sim P) \Leftrightarrow R$

$E_{14} : R \vee (P \vee \sim P) \Leftrightarrow T$

$E_{15} : R \wedge (P \wedge \sim P) \Leftrightarrow F$

$E_{16} : P \rightarrow Q \Leftrightarrow \sim P \vee Q$

$E_{17} : \sim(P \rightarrow Q) \Leftrightarrow P \wedge \sim Q.$

$E_{18} : P \rightarrow Q \Leftrightarrow \sim Q \rightarrow \sim P$

$E_{19} : P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R.$

Test whether the following is a valid argument.

Problem

(i) If Sachin hits a Centuary then he gets a free car.

(ii) Sachin hits a Centuary therefore Sachin get a free Car.

P: Sachin hits a Centuary.

Q: Sachin gets a free Car.

Sol:-

(i) $P \rightarrow Q$.

(ii) P

$\therefore$ ~~P→Q, P~~ $P, P \rightarrow Q \Rightarrow Q$ (Modus Ponens)

It is valid.

Problem:- (i) If i study then i donot fail in the Examination.

(ii) If I do not fail in the Examination then my father gift a two wheeler to me. therefore if i study then my father gifts a two wheeler to me.

P: I study

Q: I do not fail in the Examination.

R: My father gift a two wheeler to me.

Sol:-

$$\begin{array}{l} P \rightarrow Q. \\ Q \rightarrow R \\ \hline P \rightarrow R \end{array}$$

$\therefore P \rightarrow Q,\ Q \rightarrow R \Leftrightarrow P \rightarrow R$ (hypothetical syllogism)

Problem.

(i) g will become famous or i'll not become a musician.

(ii) g will become a musician therefore i'll become famous.

P : g will be come famous

Q : g will become a musician.

Sol:-

$P \vee \sim Q$

Q

$Q, P \vee \sim Q \Rightarrow P$

~~I will be~~ g will be ~~come~~ written as $\sim Q \vee P \equiv Q \rightarrow P$

$$\frac{\begin{array}{c} Q \rightarrow P \\ Q \end{array}}{P}$$

(Modus Ponens)

Problems

1. S.T R follows logically from the Premises $P \to Q$, $Q \to R$ & P.

Sol.

$\{1\}$ 1. $P \to Q$ Rule-P

$\{2\}$ 2. $Q \to R$ Rule-P.

$\{1,2\}$ 3. $P \to R$ Rule-T (1), (2) hypothetical syllogism.

$\{4\}$ 4. P. Rule-P.

$\{1,2,4\}$ 5. R. Rule-T (3)(4) Modus Ponens

(2) S.T $\sim Q,\ P \to Q \Rightarrow \sim P$.

$\{1\}$ 1. $\sim Q$ Rule-P

$\{2\}$ 2. $P \to Q$ Rule-P.

$\{1,2\}$ 3. $\sim P$ Rule-T, (1)(2) Modus tollens.

$[\sim Q,\ P \to Q \Rightarrow \sim P]$

3. S.T $\sim P$ follows logically from the Premises $\sim(P \wedge \sim Q)$, $\sim Q \vee R$, $\sim R$.

$\{1\}$ 1. $\sim(P \wedge \sim Q)$ Rule-P.

$\{2\}$ 2. $\sim P \vee Q$. Rule-T (1)

$\{3\}$ 3. $P \to Q$ Rule-T (2)

$\{4\}$ 4. $\sim Q \vee R$ Rule-P

$\{4\}$ 5. $Q \to R$ Rule-T (4)

$\{1,4\}$ 6. $P \to R$ Rule-T (3,5) hypothetical syllogism

$\{7\}$ 7. $\sim R$ Rule-P.

$\{1,4,7\}$ 8. $\sim P$ Rule-T (6)(7), Modus tollens.

4. S.T $S \vee R$ is tautologically implied by

$P \vee Q \wedge (P \to R) \wedge (Q \to S)$

Sol Given $P \vee Q$, $P \to R$, $Q \to S$.

$\{1\}$ 1. $P \vee Q$ Rule — P

$\{1\}$ 2. $\sim P \to Q$ Rule T — (1). $\sim(\sim P) \Rightarrow P$. $P \to Q \Leftrightarrow \sim P \vee Q$.

$\{3\}$ 3. $Q \to S$ Rule - P.

$\{1,3\}$ 4. $\sim P \to S$ Rule-T (2,3) $P \to Q, Q \to R \Rightarrow P \to R$.

$\{1,3\}$ 5. $\sim S \to P$ Rule-T (4) $P \to Q \Leftrightarrow \sim Q \to \sim P$ (ContraPositive)

$\{6\}$ 6. $P \to R$ Rule P.

$\{1,3,6\}$ 7. $\sim S \to R$ Rule T (5,6) $P \to Q \Leftrightarrow \sim P \vee Q$.

$\{1,3,6\}$ 8. $S \vee R$ Rule T (7). [$S \to R = \sim S \vee R$]

5. S.T $R \vee S$ follows logically from the Premises

$C \vee D$, $(C \vee D) \to \sim H$, $\sim H \to (A \wedge \sim B)$, $(A \wedge \sim B) \to R \vee S$.

$\{1\}$ 1. $(C \vee D) \to \sim H$ Rule P

$\{2\}$ 2. $\sim H \to (A \wedge \sim B)$ Rule P

$\{1,2\}$ 3. $(C \vee D) \to (A \wedge \sim B)$ Rule-T (1,2)

$\{4\}$ 4. $A \wedge \sim B \to R \vee S$ Rule-P

$\{1,2,4\}$ 5. $(C \vee D) \to R \vee S$ Rule-T (3,4)

$\{6\}$ 6. $C \vee D$ Rule - P

$\{1,2,4,6\}$ 7. $R \vee S$ Rule (5)(6)

V.Imp

6. S.T $R \wedge (P \vee Q)$ is a valid Conclusion from the Premises $(P \vee Q)$, $Q \to R$, $P \to M$ & $\sim M$.

$\{1\}$ 1. $P \vee Q$ Rule - P

$\{1\}$ 2. $\sim P \to Q$ Rule - T (1)

$\{3\}$ 3. $Q \to R$ Rule - P

$\{1,3\}$ 4. $\sim P \to R$ Rule - T (2)(3)

$\{5\}$ 5. $P \to M$ Rule - P.

$\{6\}$ 6. $\sim M$ Rule - P

$\{5,6\}$ 7. $\sim P$ Rule - T (5)(6) (Modus tollens)

$\{1,3,5,6\}$ 8. R Rule - T (4)(7) (Modus Ponens)

$\{1,3,5,6\}$ 9. $R \wedge (P \vee Q)$ Rule - T (1)(8)

7. ST $\sim R \to (S \to \sim T)$, $\sim R \vee W$, $\sim P \to S$, $\sim W \Rightarrow T \to P$.

$\{1\}$ 1. $\sim P \to S$ Rule - P

$\{1\}$ 2. $P \vee S$ Rule - T (1)

$\{3\}$ 3. $\sim R \vee W$ Rule - P

$\{3\}$ 4. $R \to W$ Rule - T (3)

$\{5\}$ 5. $\sim W$ Rule - P

$\{3,5\}$ 6. $\sim R$ Rule - T (4)(5) (Modus tollens)

$\{7\}$ 7. $\sim R \to (S \to \sim T)$ Rule - P.

$\{3,5,7\}$ 8. $S \to \sim T$ Rule - T (6)(7) (Modus Ponens)

$\{1,3,5,7\}$ 9. $\sim P \to \sim T$ Rule - T (1)(8) (hypothetical syllogism)

$\{1,3,5,7\}$ 10. $T \to P$ Rule - T (9)

~~Consistency of Premises and indirect Method of Proof.~~

Problem. ①

(i) I will get grade A in this Course or I will not graduate

(ii) If I donot graduate I will Joint army.
I got grade A ~~therefore I will not Join the~~ therefore I will not Join the army in this Course.

P: I get grade A. in this Course.

Q: I do not graduate.

R: I Join the army.

Sol:-

$$\begin{array}{l} P \vee Q \\ Q \to R \\ R \\ \hline \sim R \\ \hline \end{array}$$

$P \vee Q \Rightarrow Q \vee P.$

$\Leftrightarrow \sim Q \to P.$

$\Leftrightarrow \sim R \to \sim Q$ (Contra +ve)

$\Leftrightarrow P \to \sim R$

P

$\Rightarrow \sim R$

Problem. If I study, I will not fail in the Examination.
If I donot watch TV in the evenings, I will study.
I failed in the Examination.
Therefore I must have watched TV in the evenings.

Sol. Let P: I study

Q: I fail in the Examination.

R: I watch TV in the evenings.

The given argument reads
$$\begin{array}{r} P \to \sim Q \\ \sim R \to P \\ Q \\ \hline \therefore R \end{array}$$

The argument is logically equivalent to

$P \to \sim Q \Leftrightarrow \sim(\sim Q) \to \sim P$

$\sim R \to P$

$\Leftrightarrow \sim P \to R.$

$Q \to \sim P$

$\sim P \to R \Leftrightarrow Q \to R.$

$$\begin{array}{c} Q \\ \hline R \end{array} \qquad \begin{array}{c} Q \\ \hline R \end{array}$$

This argument is valid.

Problem: Test the validity of the following arguments

(i)
$$\begin{array}{l} P \wedge Q \\ P \to (Q \to R) \\ \hline \therefore R \end{array}$$

Sol: $\therefore P \wedge Q$ is true both P & Q are true

P is true & $P \to (Q \to R)$ is true

$Q \to R$ has to be true

$\therefore Q$ is true & $Q \to R$ is true, R has to be true

Hence the given argument is valid.

(ii)

$$\begin{array}{l} P \to R \\ Q \to R \\ \hline \therefore (P \vee Q) \to R. \end{array}$$

Sol

$(P \to R) \wedge (Q \to R) \Leftrightarrow (\sim P \vee R) \wedge (\sim Q \vee R)$

$\Leftrightarrow (R \vee \sim P) \wedge (R \vee \sim Q)$ by Commutative law.

$\Leftrightarrow R \vee (\sim P \wedge \sim Q)$ by distributive law

$\Leftrightarrow R \vee \sim (P \vee Q)$ by Demorgan's law

$\Leftrightarrow \sim (P \vee Q) \vee R$ by Commutative law.

$\Leftrightarrow (P \vee Q) \to R.$

This logical Equivalence shows that the given argument is valid.

Problem:

$$\begin{array}{l} P \to Q \\ R \to S \\ P \vee R \\ \hline \therefore Q \vee S \end{array}$$

Sol:

$(P \to Q) \wedge (R \to S) \wedge (P \vee R)$

If a ~~body~~ baby is hungry, then ~~the~~ baby cries. If the baby is not mad, then he does not cry. If a ~~bab~~ baby is mad, then he has a red face. Therefore, If a baby is hungry, then he has a red face.

P: A baby is hungry

Q: A baby Cries

R: A baby is mad and

S: A baby has a red face.

$$\begin{array}{l} P \to Q \\ \sim R \to \sim Q \\ \underline{R \to S} \\ \therefore\ P \to S \end{array}$$

Ag: $P \to Q \equiv \sim P \vee Q$

$\sim R \to \sim Q$ is the Contrapositive of $Q \to R$

$$\begin{array}{l} \therefore P \to Q \\ Q \to R \\ \underline{R \to S} \\ \underline{P \to S} \end{array}$$

If a pair of angles A & B are right angles, then they are Equal. The angles A & B are Equal. Hence the angles A.

Problem - S.T $\sim S$ is a valid argument from the Premises $P \to Q$, $(\sim Q \vee R) \cap (\sim R)$, $\sim(\sim P \wedge S)$

{1} 1. $P \to Q$ Rule P

{2} 2. $(\sim Q \vee R) \wedge (\sim R)$ Rule P

{2} 3. $\sim Q \vee R$ Rule-T Simplification (2)

{2} 4. $Q \to R$ Rule T $P \to Q \Leftrightarrow \sim P \vee Q$ (3)

{1,2} 5. $P \to R$ Rule-T (1)(4), $P \to Q, Q \to R \Leftrightarrow P \to R$

$\{6\}$ 6. $\sim(\sim P \wedge S)$ Rule P.

$\{6\}$ 7. $\sim(\sim P \wedge \sim\sim S)$ Rule T $\sim\sim S = S$. $\sim(P \to Q) \Leftrightarrow P \cap \sim Q$.

$\{6\}$ 8. $\sim(\sim(\sim P \to \sim S))$ Rule T (7).

$\{6\}$ 9. $\sim(\sim(\sim S))$ Rule T $\sim(P \to Q) \Rightarrow \sim Q$ (8)

$\{6\}$ 10. $\sim S$ Rule T $\sim\sim S = S$ (8) (Doubt)

Consistency of Premises and indirect method of Proof

$H_1, H_2, \ldots H_m$ is said to be consistent if the conjunction has the truth value T for some assignment of the truth values to the atomic variables appearing in $H_1, H_2 \ldots H_m$.

A set of formulas $H_1, H_2, \ldots H_m$ is said to be inconsistent if the $H_1 \wedge H_2 \wedge \ldots \wedge H_m \equiv R \wedge \sim R$. The trueth value is F.

Indirect method of Proof

The notion of inconsistency is used in a procedure called proof by contradiction or indirect method of proof.

The technique of indirect method of proof is as follows.

1) In order to show that a conclusion 'c' follows logically from the premises $H_1, H_2, H_3 \ldots H_m$. we assume that 'c' is false and consider $\sim c$ as an additional premises.

2) If the new set of premises is inconsistent then they imply contradiction, therefore 'c' is true whenever $H_1 \wedge H_2 \wedge H_3 \wedge \ldots \wedge H_m$ is T.

(3) Then 'c' follows logically from the Premises $H_1, H_2, H_3, \ldots H_m$.

Problem. Prove by indirect method $\sim Q, P \to Q, P \vee R \Rightarrow R$.

Sol.

$\{1\}$	1.	$\sim R$	(Assumption).
$\{2\}$	2.	$P \vee R$	Rule-P.
$\{1,2\}$	3.	$\sim P \to R$	Rule-T (2)
$\{2\}$	4.	$\sim R \to P$	Rule-T (3) Contrapositive
$\{1,2\}$	5.	P	Rule-T (1) (4)
$\{6\}$	6.	$P \to Q$	Rule-P
$\{1,2,6\}$	7.	Q	Rule T (5) (6)
$\{8\}$	8.	$\sim Q$	Rule-P
$\{1,2,6,8\}$	9.	$Q \wedge \sim Q$	Rule-T (7) (8)

$Q \wedge \sim Q$ is F, our assumption is false, hence R follows logically from the given premises.

Problem By using indirect method S.T

$P \to Q,\ Q \to R,\ P \vee R \Rightarrow R.$

Sol

{1}	1.	$\sim R$	Assumption.
{2}	2.	$P \vee R$	Rule-P
{2}	3.	$\sim P \to R$	Rule-T (2)
{2}	4.	$\sim R \to P$	Rule-T (2) Contrapositive.
{1,2}	5.	P	Rule-T (1)(4)
{6}	6.	$P \to Q$	Rule-P
{1,2,6}	7.	Q	Rule-T (5)(6) (Modus Ponens)
{8}	8.	$Q \to R$	Rule-P.
{1,2,6,8}	9.	R	Rule-T (7)(8) (Modus Ponens)
{1,2,6,8}	10.	$R \wedge \sim R$	Rule-T (9)

$R \wedge \sim R$ is F, our assumption is false, hence R follows logically from the given premises.

Problem S.T the following set of Premises are inconsistent

$P \to Q,\ P \to R,\ Q \to \sim R,\ P$.

{1}	1. $P \to Q$	Rule-P	
{2}	2. P	Rule-P.	
{1,2}	3. Q	Rule-T	
{4}	4. $Q \to \sim R$	Rule-P	
{2,4}	5 $\sim R$	Rule-T	(3)(4)
{6}	6. $P \to R$	Rule-P	
{6}	7. $\sim R \to \sim P$	Rule-T	(6) ContraPositive.
{1,2,4}	8. $\sim P$	Rule-T	(5)(7)
{1,2,4,6}	9. $P \wedge \sim P$	Rule-T	(2)(8)
{1,2,4,6}			

Problem S.T $\sim(P \wedge Q)$ follows logically from $\sim P \wedge \sim Q$

→

{1}	1. $\sim(\sim(P \wedge Q))$	Additional Premises
{1}	2. $P \wedge Q$	Rule T (1)
{1}	3. P or Q	Rule T (2)
{4}	4. $\sim P \wedge \sim Q$	Rule P
{4}	5. $\sim P$ or $\sim Q$	Rule-T (4)
{1,4}	6. $P \wedge \sim P$	Rule-T (3)(5)

$P \wedge \sim P$ is Contradiction, our assumption is F.

Hence $\sim(P \wedge Q)$ follows logically from $\sim P \wedge \sim Q$.

Problem show that the following are inconsistent

(i) If Jack misses many classes through illness then he fails highschool.

(ii) If Jack fails highschool then he is uneducated.

(iii) If Jack reads a lot of books then he is not uneducated.

(iv) Jack misses many classes through illness and reads a lot of books.

(i) $P \rightarrow Q$. (ii) $Q \rightarrow R$ (iii) $S \rightarrow \sim R$ (iv) $P \wedge S$.

$\{1\}$	1. $P \rightarrow Q$	Rule-P	
$\{2\}$	2. $Q \rightarrow R$	Rule-P.	
$\{1,2\}$	3. $P \rightarrow R$	Rule-T	(1)(2)
$\{4\}$	4. $S \rightarrow \sim R$	Rule-P	
$\{4\}$	5. $R \rightarrow \sim S$	Rule-T	Contrapositive (4)
$\{1,2,4\}$	6. $P \rightarrow \sim S$	Rule-T	(3) (5)
$\{7\}$	7. $P \wedge S$	Rule-P	
$\{7\}$	8. P	Rule-T	(7)
$\{7\}$	9. S	Rule-T	(7)
$\{1,2,4,7\}$	10. $\sim S$	Rule-T	(6) (8)
$\{1,2,4,7\}$	11 $S \wedge \sim S$	Rule-T	(9) (10)

$S \wedge \sim S$ is a contradiction, our assumption is F.

The Predicate logic

Let us Casider the statement: Ravi is a bachelor.
is a bachelor is called as Predicate ~~Ra~~, Ravi is Subject. In symbolic form, it can be written as $B(r)$

Predicates are denoted by Capital letters and name of individual objects are denoted by Small letters. A Predicate requires m $(m>0)$ names or objects is called an m-place Predicate

Example: $\underbrace{\text{Ravi}}_{S}$ $\underbrace{\text{is taller than}}_{P}$ $\underbrace{\text{Venkat}}_{S}$

It is a 2-place Predicate because it is related to 2 subjects. ∴ The statement can be written as Symbolic form $T(r, v)$

$\underbrace{\text{Seetha}}_{S}$ $\underbrace{\text{sits between}}_{P}$ $\underbrace{\text{Jyothi}}_{S}$ and $\underbrace{\text{Meena}}_{S}$

It is a 3-place Predicate. It is written as $S(s, l, m)$

Note:- That the order in which the names or objects appear in the statement as well as in the predicate is important.

Quantifiers

There are two types of Quantifiers.

1) universal Quantifiers and ($\forall$)

(2) Existential Quantifiers ($\exists$)

A PrePosition involving the universal or the existential Quantifier is called Quantifying Statement

The symbol (x) or $(\forall x)$ is called universal Quantifier. The symbol $(\forall)$ has been used to denote the Phrase for all. "for Every" "for Each". "Everything" Quantifier. The Symbol $(\exists x)$ is called Existential Quantifier. The Symbol $(\exists)$ has been used to denote the Pharase "for some", "for atleast one", "there Exists".

Example :- All roses are Red.

If x is a rose then x is Red / let $R(x)$: x is a Rose $P(x)$: x is Red.

Symbolic form :

$$(\forall x)\, R(x) \rightarrow P(x)$$

Every apple is Red

If x is an apple then x is Red. / let

Predicates

Let us consider the following two statements.

1) Radha is a girl —①

(ii) Seeta is a girl —②

Radha (or) Seeta is the Subject of the statement.

The second part "is a girl" which refers to a property that the Subject can have is called the Predicate.

We symbolize a Predicate by a Capital letter and the name of individuals or objects in general by lower case letters.

The Predicate "is a girl" is denoted by G.

Radha by (r) and Seeta by "s"

The statement "Radha is a girl" is denoted by G(r).

"Seeta is a girl" is denoted by G(s).

Any statement of the type "p is Q".

where Q is a Predicate and p is the Subject.

Can be denoted by Q(P)

A Predicate requiring m (m>0) names is called an m-place Predicate.

Equations ① & ② are 1-place Predicates. A statement is called a 0-place Predicate when no names are associated with the statement.

Example :- 1) Radha is taller than Seeta.

If "T" denotes "is taller than" then T is a two- Place Predicates.

(2) "Rita sits between Nita & sita" is a three -place Predicate.

(3) Radha & Seeta Played bridge (Caroms) against Rita & Nita. is a four place Predicate.

~~Till now~~

The Predicate logic

let us Consider the statement.

Ravi is a bachelor.

"is a bachelor" is called as a Predicate.

Symbolically we denote it as letter B

Ravi is subject and denoted it as r.

It can be written as B(r)

In general, Any statement of the form "p is Q" where Q is a Predicate & p is the Subject can be denoted by Q(P)

A Predicate requires m (m>0) names or objects is called an m-place Predicate.

Example. Indu is a student

"is a student" is 1-Place Predicate because, it is replaced to one object (or) Indu.

This statement Can be written as $S(i)$

Note :- Predicates are denoted by Capital letters & name of individuals objects are denoted by Small letters

Example. Ram is taller than venkat.

$\underbrace{\text{Ram}}_{\text{object}}$ $\underbrace{\text{is taller than}}_{\text{Predicate}}$ $\underbrace{\text{venkat}}_{\text{object}}$

"is taller than" is a 2-Place Predicate because it is reduced to 2 objects. This statement Can be written as

$T(r, v)$

Problems Consider the following statements.

$P(x)$: $x \leq 3$ $Q(x)$: $x+1$ is odd $R(x)$: $x > 0$.

Write down the truth values of the following

(i) $P(2)$

$P(x)$; $x \leq 3$

$P(2)$: $2 \leq 3$ (False). which is false.

(ii) $\sim Q(4)$.

$Q(x)$: $x+1$ is odd

$Q(4) = 4+1$ is odd. which is true.

$\therefore \sim Q(4)$ is false.

(iii) $P(-1)$ is the Proposition "$-1 \leq 3$" which is true and $Q(1)$ is the Proposition "$1+1$ is odd which is false.

$P(x): x \leq 3$

$P(-1): -1 \leq 3.$ which is true.

$Q(x): x+1$ is odd.

$Q(1) = 1+1.$ is odd which is false.

$\left.\begin{matrix} P(-1) \wedge Q(1) \\ (T \wedge F) \end{matrix}\right\} = \text{False}.$

$\therefore P(-1) \wedge Q(1)$ is false

(iv) $\sim P(3) \vee R(0)$

$P(x): x \leq 3$

$P(3): 3 \leq 3$ which is true

$\sim P(3):$ which is False.

$R(x): x > 0.$

$R(0): 0 > 0$ which is false.

$\sim (P(3) \vee R(0)$ is false.

$(F \vee F)$

(v) $P(0) \rightarrow Q(0)$

$P(x): x \leq 3$

$P(0): 0 \leq 3$ which is true.

$Q(x): x+1$ is odd

$Q(0): 0+1$ is odd which is true.

$P(x) \rightarrow Q(x)$

$T \rightarrow T$

$P(0) \rightarrow Q(0)$ is true.

(vi) $P(1) \Leftrightarrow \sim Q(2)$

$P(1)$ is True & $Q(2)$ is True.

$P(1) \Leftrightarrow Q(2)$ is false.

(vii) $P(4) \vee (Q(1) \wedge R(2))$

$P(x): x \leq 3$

$P(4): 4 \leq 3$ False.

$Q(x): x+1$ is odd

$Q(1): 1+1$ is false.

$R(x): x > 0$

$R(2): 2 > 0$ is True.

$\therefore P(4) \vee (Q(1) \wedge R(2))$.

[$Q(1) = F$ $F \wedge T = F$]

$R(2) = T$

$Q(1) \wedge R(2)$ is False

$P(4) \vee$ False.

$F \vee F = F$

$\therefore P(4) \vee (Q(1) \vee R(2))$ is false.

V(ii) $P(2) \wedge (Q(0) \vee \sim R(2))$

$P(x): x \leq 3$

$P(2): 2 \leq 3$ (True).

$Q(x): x+1$ is odd

$Q(0) = 0+1$ is odd (True)

$R(x): x > 0$

$R(2): 2 > 0$ is True.

$\sim R(2)$: False

$\therefore Q(0) \vee \sim R(2)$

$T \vee F$ i.e true (T)

So $P(2) \wedge (Q(0) \vee \sim R(2))$

$T \wedge T = T$

$\therefore P(2) \wedge (Q(0) \vee \sim R(2))$ is true.

Note:- That the order in which the names or objects appear in the statement as well as in the predicate is important.

Example. Phaneendhar is taller than Mohan.

the ~~pho~~ Predicate "is taller than" is a 2-place Predicate.

Given statement in symbolic form is $T(P, m)$

T: is taller than

p: phanendhar

m: Mohan.

(i) Suman sits between Rahul and Nithin.

let us consider

S: sits between.

~~s~~: suman

~~r~~: Rahul.

~~n~~ = Nithin

Symbolic ~~S(s, r, n)~~ $S(s, r, n)$

(ii) Venkat & Karthik played (Commons) bridge against Rowith & Siva.

P: Played bridge against.

V: venkat

K: Karthik.

R: Rowith

S: Siva.

$P(v, k, r, s)$.

Note: An n- place Predicate requires n names of objects to be inserted in fixed Positions in order to obtain a statements. The Position of these names or objects is importance. If 'S' is an n-place letter $a_1, a_2, \ldots a_n$ are the names of objects. then $S(a_1, a_2, \ldots a_n)$ is a statement.

Quantifiers:-

The phrase which indicates the Quantity is called a Quantifier.

The Quantifiers are $\forall$ (for all) & $\exists$ (there exists)

$\forall x$, $P(x)$ means the predicate P is ~~said to~~ satisfied for all x.

$\exists x$, $P(x)$ " " " " " " for some x

Quantified statement :- A Proposition involving Quantifier is called the Quantified statement.

There are two types of a Quantifier

Universal Quantifier :- $\forall$ is called universal Quantifier.

Existential Quantifier :- $\exists$ is called the existential Quantifiers.

Free Variable :- The variables, which are not bounded by the Quantifiers are called the free variables.

Example. $P(x)$ & $Q(x)$, here x is a free variable.

Bounded Variable :- The variables, which are bounded by Quantifiers are called the bounded variables.

Example: $\exists\, x \in z$, $P(x)$, $\forall\, x \in z$, $r(x)$

Here x is a bounded variables.

Truth set :- The set of truth values of a Predicate $P(x)$ is Called the truth set & written as $T(P)$

Negation of a Quantified statement:

To find the negation of a Quantified statement, Change the Quantifier from universal to Existential & vice versa.

$$\sim(\forall x, P(x)) \equiv \exists x, (\sim P(x))$$

$$\sim(\exists x, P(x)) \equiv \forall x (\sim P(x))$$

Note $\forall x, \forall y \; P(x,y) \equiv \forall y \; \forall x (P(x,y))$

$$\exists x, \exists y \; P(x,y) \equiv \exists y \; \exists x (P(x,y))$$

$$P \wedge \exists x \, Q(x) \equiv \exists x (P \wedge Q(x))$$

$$P \vee \forall x \, Q(x) \equiv \forall x (P \vee Q(x))$$

$$\exists x, P(x) \equiv \sim \forall x \sim P(x)$$

$$\forall x, P(x) \equiv \sim \exists x \sim P(x)$$

Quantifiers:

Example: All Roses are Red.

If x is a Rose, then x is red.

Let us denote $R(x)$: x is a Rose.

& $P(x)$: x is a red.

then it Can write as $(x)(R(x) \rightarrow P(x))$

$(x)(R(x)) \rightarrow P(x)$ is also written as

$\forall (x)(R(x) \rightarrow P(x))$

The symbols (x) or $(\forall x)$ are called universal Quantifiers

Example: (1) All men are mortal.

Let us Parapharse these in the following manner.

For all x, if x is a man, then x is a mortal.

Let us Consider statement nos (numbers) are as follows:

$M(x)$: x is a man, $H(x)$: is a Mortal.

$(x)(M(x) \rightarrow H(x))$

(2) Every apple is red.

For all x, If x is an apple then x is red.

$A(x)$: x is an apple, $R(x)$: x is a red.

$(x)(A(x) \rightarrow R(x))$

(3) Any integer is Positive or −ve.

for all x, if x is an integer, then x is either +ve or −ve.

$Z(x)$: x is an integer $P(x)$: x is either +ve or −ve.

$(x)\ (Z(x) \rightarrow P(x))$

note :- The statement $(x)\ (M(x))$ can be translated as

$\left\{\begin{array}{l}\text{for all } x,\ x \text{ is a man.}\\ \text{for every } x,\ x \text{ is a man}\\ \text{Every thing is a man}\\ \text{for Each } x,\ x \text{ is a man}\end{array}\right.$

The symbol "$(\exists x)$" called the existential Quantifier.

The symbol "$\exists$" has been used to denote the Phrases "for some" & "there exits" "for at least one".

A Proposition involving the universal or the existential Quantifier is called a Quantified statement

For the universe of all integers, let

$P(x)$, $x > 0$, $Q(x)$: x is Even, $R(x)$: x is a Perfect Square

$S(x)$: x is divided by 3, $T(x)$: x is divisible by 7.

Write down the following Quantified statements in Symbolic form.

(i) At least one integer is Even.

$(\exists x), Q(x)$

(ii) There exists a +ve integer that is Even.

$(\exists x)(P(x) \wedge Q(x))$

Q : Even integer
S : divisible by 3.

(iii) Some even integers are divisible by 3.

$(\exists x)(Q(x) \wedge S(x))$.

x : integer
Q : Even integer

(iv) Even integer is either Even or odd.

$\forall(x)(Q(x) \vee \sim Q(x))$.

(v) If x is even & a perfect square then x is not divisible by 3.

Q : Even.
R : Perfect Square
S : divisible by 3

$\forall(x)((Q(x) \wedge R(x)) \to \sim S(x))$

(vi) If x is odd or is not divisible by 7, then x is divisible by 3.

Q : odd
T : divisible by 7.
S : divisible by 3.

$\forall x((\sim Q(x)) \vee \sim T(x)) \to S(x)$

Inference theory of Predicate Calculus

Rule US: universal specification (or) instantilation.

$(x)\ A(x) \Leftrightarrow A(y)$ From $(x)A(x)$, one can conclude $A(y)$

Rule ES: Existential Specification.

$(\exists x)\ A(x) \Leftrightarrow A(y)$

Rule EG:- Existential generalization

$A(x) \Leftrightarrow (\exists y)\ A(y)$

Rule UG: universal generalization

$A(x) \Leftrightarrow (y)\ A(y)$.

Equivalence Formulas

Two Quantified statements are said to be logically equivalent whenever they have the same truth values in all possible situations.

1) $\forall x\ [P(x) \wedge Q(x)] \Leftrightarrow (\forall x,\ P(x)) \wedge (\forall x\ Q(x))$
2) $\exists x\ [P(x) \vee Q(x)] \Leftrightarrow (\exists x\ P(x)) \vee (\exists x\ Q(x))$
3) $\exists x\ [P(x) \rightarrow Q(x)] \Leftrightarrow \exists x\ [\sim P(x) \vee Q(x)]$
4. $\sim [(\exists x)\ A(x)] \Leftrightarrow (x) \sim A(x)$
5. $\sim ((x)\ A(x)) \Leftrightarrow (\exists x) \sim A(x)$

Rules for negation

$\sim[\forall x, P(x)] = \exists x\ [\sim P(x)]$

$\sim[\exists x, P(x)] \equiv \forall x\ [\sim P(x)]$

1. Verify the validity of the following argument.

1. All men are mortal. Socrates is a man. Therefore, socrates is a mortal.

(or) S.T $(x)\left[H(x) \to M(x)\right] \wedge H(s) \Rightarrow M(s)$

Let $H(x)$: x is a Man

$M(x)$: x is mortal.

S : Socrates.

we have to S.T $(\forall x)\ (H(x) \to M(x)) \wedge H(s) \Rightarrow M(s)$

$\{1\}$ 1. $(\forall x)\ (H(x) \to M(x))$ Rule P

$\{1\}$ 2. $H(s) \to M(s)$ US (1)

$\{3\}$ 3. $H(s)$ Rule P.

$\{1,3\}$ 4. $M(s)$ Rule T (2) (3), $P, P \to Q \Rightarrow Q$ Modus Pones

2. S.T $(x)\ (P(x) \to Q(x)) \wedge (x)\ (Q(x) \to R(x)) \Rightarrow (x)\ P(x) \to R(x)$

$\{1\}$ 1. $(x)\ (P(x) \to Q(x))$ Rule P

$\{1\}$ 2. $P(y) \to Q(y)$ US, (1)

$\{3\}$ 3. $(x)\ (Q(x) \to R(x))$ Rule P.

$\{3\}$ 4. $Q(y) \to R(y)$ US (3)

$\{1,3\}$ 5. $P(y) \to R(y)$ Rule T "$P \to Q,\ Q \to R \Rightarrow P \to R$" Rules of syllogism.

$\{1,3\}$ 6. $(x)\ (P(x) \to R(x))$ UG (5).

Rule P : A Premise may be introduced at any Point in the derivation.

Rule T :- A formula 'S' may be introduced in a deviation if 'S' is tautologically implied by one or more of the Preceding formulas in the deviation.

Prob : All integers are rational number. Some integers are Powers of 2. therefore, some rational numbers are Powers of 2".

Let $I(x)$: x is an integer.

$R(x)$: x is a rational number

$P(x)$: x is a Power of 2.

$\cancel{(x)\ I(x) \to R(x) \wedge (\exists x)\ I(x) \wedge P(x)} \Rightarrow (\exists x)\ (R(x) \wedge P(x))$

$\cancel{\{1\}\ 1.\ \exists x\ (I(x) \wedge P(x))}$ ~~Rule P~~

$\cancel{\{1\}\ 2.\ I(y) \wedge P(y)}$

$\forall(x)\ (I(x) \to R(x)).$

$(\exists x)\ (I(x) \wedge P(x)) \Rightarrow (\exists x)\ (R(x) \wedge P(x)).$

$\{1\}$	1.	$(\forall x)\ (I(x) \to R(x))$	Rule P.
$\{2\}$	2.	$(\exists x)\ (I(x) \wedge P(x))$	Rule P.
$\{1\}$	3.	$I(y) \to R(y)$	Rule US (1)
$\{2\}$	4.	$I(y) \wedge P(y)$	Rule ES (2)
$\{2\}$	5.	$I(y)$	Rule T (4)
$\{1,2\}$	6.	$R(y)$	Rule T (3)(5) Modus Ponens

$\{2\}$ 7. $P(y)$ Rule T (4)

$\{1,2\}$ 8. $R(y) \wedge P(y)$ Rule T (6) (7)

$\{1,2\}$ 9. $(\exists x)(R(x) \wedge P(x))$ Rule EG.

5. All fathers are Males.
Some students are fathers.
$\therefore$ Some student are males.

Sol

$F(x)$: x is father

$M(x)$: x is male.

$S(x)$: x is student.

$\forall(x)\ (F(x) \to M(x))$

$\exists(x)\ (S(x) \wedge F(x)) \Rightarrow (\exists x)\ (S(x) \wedge M(x))$

$\{1\}$ 1. $(\forall x)(F(x) \to M(x))$ Rule P

$\{1\}$ 2. $F(y) \to M(y)$ Rule US (1)

$\{3\}$ 3. $(\exists x)\ (S(x) \wedge F(x))$ Rule P.

$\{3\}$ 4. $S(y) \wedge F(y)$ Rule ES (1)

$\{3\}$ 5. $F(y)$ Rule - T (4)

$\{1,3\}$ 6. $M(y)$ Rule - T (2) (5)

$\{3\}$ 7. $S(y)$ Rule - T (4)

$\{1,3\}$ 8. $S(y) \wedge M(y)$ Rule - T (7) (6)

$\{1,3\}$ 9. $(\exists x)(S(x) \wedge M(x))$ Rule EG.

6. Some dogs are animals.

Some cats are animals.

$\therefore$ Some dogs are cats.

$D(x)$: x is a dog

$A(x)$: x is animals.

$C(x)$: x is a cat.

$(\exists x)(D(x) \wedge A(x))$

$(\exists x)(C(x) \wedge A(x)) \Rightarrow (\exists x)(D(x) \wedge C(x))$

$\{1\}$ 1. $(\exists x)(D(x) \wedge A(x))$ Rule P.

$\{1\}$ 2. $D(y) \wedge A(y)$ Rule ES (1)

$\{2\}$ 3. $(\exists x)(C(x) \wedge A(x))$ Rule P

$\{2\}$ 4. $C(y) \wedge (A(y))$ Rule ES(3)

$\{1\}$ 5. $D(y)$ Rule T(2)

$\{2\}$ 6. $C(y)$ Rule T (4)

$\{1,2\}$ 7. $D(y) \wedge C(y)$ Rule T (5) (6)

$\{1,2\}$ 8. $(\exists x)(D(x) \wedge C(x))$ Rule EG.

7. $(\exists x) M(x)$ follows logically from the Premises

$(\forall x)(H(x) \rightarrow M(x))$, $(\exists x) H(x)$

$\{1\}$ 1. $(\forall x)(H(x) \rightarrow M(x))$ Rule-P

$\{1\}$ 2. $H(y) \longrightarrow M(y)$ Rule US(1)

$\{3\}$ 3. $(\exists x) H(x)$ Rule P

$\{3\}$ 4. $H(y)$ Rule ES(3)

$\{1,3\}$ 5. $M(y)$ Rule T (2) (4)

$\{1,3\}$ 6. $(\exists x) M(x)$ Rule EG.

8\. All Men are fallible.

All kings are Men.

$\therefore$ All Kings are fallible.

$\rightarrow$ $M(x)$: x is Men

$F(x)$: x is fallible

$K(x)$: x is kings.

$\forall(x)$: $(M(x) \rightarrow f(x))$

$\forall(x)$; $(K(x) \rightarrow M(x)) \Rightarrow \forall(x)\,(K(x) \rightarrow f(x))$

$\{1\}$	1.	$\forall(x)\,(M(x) \rightarrow F(x))$	Rule P
$\{1\}$	2.	$M(y) \longrightarrow f(y)$	Rule US
$\{3\}$	3.	$\forall(x)\,(K(x) \rightarrow M(x))$	Rule P
$\{3\}$	4.	$K(y) \longrightarrow M(y)$	Rule US.
$\{1\}$	5.	$f(y) \longrightarrow M(y)$	Rule T (1) Converse.
$\{3\}$	6.	$M(y) \longrightarrow K(y)$	Rule T (2) Converse.
$\{1, 3\}$	7.	$f(y) \longrightarrow K(y)$	Rule T (5)(6) hypothetical syllogism.
$\{1, 3\}$	8.	$K(y) \longrightarrow f(y)$	Rule T (7) Converse.
$\{1, 3\}$	9.	$(\forall(x))\,(K(x) \rightarrow f(x))$	Rule UG.

9. Every living thing is a plant or an animal.
David's dog is alive and it is not a plant.
All animals have heart.
∴ David's dog has a heart.

$P(x)$: x is a plant.
$A(x)$: x is an animal.
$H(x)$: x have a heart.
$D(x)$: David's Dog.

$(\forall(x))\ (P(x) \vee A(x))$.

$\sim P(d)$.

$\forall(x)\ (A(x) \rightarrow H(x)) \Rightarrow H(d)$.

$\{1\}$	1.	$\forall x\,(P(x) \vee A(x))$	Rule P.
$\{1\}$	2.	$P(D) \vee A(D)$	Rule US (1)
$\{3\}$	3.	$\sim P(D)$	Rule P.
$\{2,3\}$	4.	$A(D)$	Rule T (2)(3)
$\{5\}$	5.	$(\forall(x))\,(A(x) \rightarrow H(x))$	Rule P.
$\{5\}$	6.	$A(D) \rightarrow H(D)$	Rule US (5)
$\{1,3,5\}$	7.	$H(D)$	Rule T (4)(6)

Write the following statements in Symbolic form.

i) Something is good.

we denote $G(x)$: x is good.

"There is atleast one x such that x is good."

$(\exists x)\,(G(x))$

(ii) Every thing is good

" for all x, x is good".

$(\forall x)\,(G(x))$

(iii) Nothing is good

" for all x, x is not good".

$(\forall x)\,(\sim G(x))$

(iv) Something is not good.

" ~~for all~~ there is atleast one x such that x is not good.

$(\exists x)\,(\sim G(x))$

Write each of the following in symbolic form.

(a) All men are good

let $M(x)$: x is a men
$G(x)$: x is good.

" for all x, if x is a man, then x is good.

$\forall(x)\,(M(x) \rightarrow G(x))$

(b) No men are good

" for all x, if x is a man, then x is not good."

$(\forall x) [H(x) \rightarrow \sim G(x))$

(c) Some men are good

" there is an x, such that x is a man & x is good."

$(\exists x) (H(x) \wedge G(x))$

(d) Some men are not good

" there is an x, such that x is a man & x is not good"

$(\exists x) (H(x) \wedge \sim G(x))$

Translate each of the following statements into symbols, using Quantifiers, Variables & Predicate Symbols.

(a) All birds can fly.

$B(x)$: x is a bird

$F(x)$: x can fly

" for all x, if x is a bird then x can fly.

$\forall(x) (B(x) \rightarrow F(x))$

(b) Not all birds can fly.

$\sim [\forall(x), B(x) \rightarrow F(x)]$

$\sim [\forall(x), \sim B(x) \vee F(x)]$

$\exists x, [B(x) \wedge \sim F(x)]$

$\sim (P \rightarrow Q)$
$\sim (\sim P \vee Q)$
$P \wedge \sim Q$.

(c) All babies are illogical.

$\forall(x)\,[B(x) \to I(x)]$.

(d) Some babies are illogical

$(\exists x)\,[B(x) \wedge I(x)]$

(e) All mathematician are rational.

$M(x)$: x is a mathematician

$R(x)$: x is rational

$\forall(x),\,[M(x) \to R(x)]$

(f) Some men are giants.

$(\exists x)\,[M(x) \wedge G(x)]$

(g) Some mens are not giants.

$(\exists x)\,[M(x) \wedge \sim G(x)]$

(h) All men are giants.

$\forall(x)\,[M(x) \to G(x)]$

(i) No men are gaints.

$\sim[\forall(x)\,[M(x) \to G(x)]]$

(J) There is a student who likes Mathematics but not history.

$\exists x\,[S(x) \wedge M(x) \wedge \sim H(x)]$

k) x is an odd integer & x is Prime.

$O(x) \wedge P(x)$

l) For all integers x, x is odd & x is Prime.

$\forall(x)[O(x) \wedge P(x)]$

(m) For each integer x, x is odd and x is Prime.

$\forall(x)[O(x) \wedge P(x)]$

n) There is an integer x such that x is odd and x is Prime.

$\exists(x)[O(x) \wedge P(x)]$

o) Some numbers are rational

$\exists(x)[N(x) \wedge R(x)]$.

(P) not all numbers are rational.

$\sim[\forall(x), N(x) \rightarrow R(x)]$

(Q) $\exists(x)[N(x) \wedge \sim R(x)]$

(Q) Not Every Graph is planar.

$\sim[\forall(x), G(x) \rightarrow P(x)]$

Note "all true" $[\forall x, F(x)] \equiv [\sim(\exists x, \sim F(x)]$ "none False".

"all false $\{\forall(x), [\sim F(x)]\} \equiv \{\sim[\exists x, F(x)]\}$ "none True"

"not all true" $\{\sim[\forall x, F(x)] \equiv \{\exists x, \sim F(x)]]$ "at least one false".

"not all false $\{\sim(\forall x, \sim F(x)]] \equiv [\exists x, F(x)]$ "at least one true".

let $P(x)$: $x^2-7x+10=0$

$Q(x)$: $x^2-2x-3=0$

$R(x)$; $x<0$

Determine the truth or falsity of the following statements when the universe U contains only the integers 2 & 5.

(i) $\forall x, P(x) \to \sim R(x)$.

The universe is $U=\{2,5\}$.

$x^2-7x+10 \equiv (x-5)(x-2)$

$P(x)$ is true for $x=5$ & 2.

i.e $P(x)$ is true for all $x \in U$.

$x^2-2x-3=0. \Rightarrow$

$x^2-2x-3=(x-3)(x+1)$

$\therefore$ $Q(x)$ is true only for $x=3$ & $x=-1$.

$\because x=3$ & $x=-1$ are not in the universe.

$Q(x)$ is false $\forall x \in U$.

$R(x)$ is false $\forall x \in U$. [$\because$

$\because$ $P(x)$ is true for all $x \in U$.

$\sim r(x)$ is true for all $x \in U$, the statement $\forall x$.

$P(x) \to \sim R(x)$ is true

[$x^2-2x-3=0$

$x^2-3x+x-3=0$

$x(x-3)+1(x-3)=0$

$x=3,-1$ $\{2,5\}$

$-1 \neq \{2,5\}$

only $3 = \{2,5\}$]

(ii) $\forall x, Q(x) \rightarrow R(x)$

$Q(x)$ & $R(x)$ are false for $x = 2$.

the statement $\exists x, Q(x) \rightarrow R(x)$ is true

(iv) $\exists x, P(x) \rightarrow R(x)$

$\because$ $P(x)$ is true, $\forall x \in U$ but $R(x)$ is false $\forall x \in U$

the statement. $P(x) \rightarrow R(x)$ is false $\forall x \in U$.

$\exists x, P(x) \rightarrow R(x)$ is false.

Negate & simplify each of the following

$\exists x, [P(x) \vee q(x)]$

By using the rule of negation for Quantified statements and the law of logic.

$$\sim [\exists x, P(x) \vee q(x)] \equiv \forall x, [\sim P(x) \vee q(x)]$$
$$\equiv \forall x, [\sim P(x) \wedge q(x)]$$

(ii) $\forall x, [P(x) \wedge \sim q(x)]$

$$\sim \{\forall x\, (P(x)) \wedge \sim q(x))] \equiv \exists x, [\sim \{ P(x) \wedge \sim q(x)]$$
$$\equiv \exists x\, [\sim P(x) \vee q(x)]$$

(iii) $\forall x\, [P(x) \rightarrow Q(x)]$

$$\sim \{\forall x, [P(x) \rightarrow Q(x)] \equiv \exists x\, [\sim (P(x)) \rightarrow Q(x)]$$
$$\equiv \exists x\, (P(x) \wedge \sim Q(x)]$$

write down the following Proposition in symbolic form and find its negation

" All integers are rational numbers and some rational numbers are not integers.

$P(x)$: x is a rational number

$Q(x)$: x is an integer.

Z : Set of all integers

Q : set of all rational numbers.

in symbolic form $\{\forall x \in Z, P(x)\} \wedge \{\exists x \in Q, \sim Q(x)\}$

negation $\{\exists x \in Z, \sim P(x)\} \vee \{\forall x \in Q, Q(x)\}$.

" Some integers are not rational numbers or Every rational number is an integer "

The universal Quantifiers was used to translate expressions such as " for all " " Every " and " for any " $\boxed{\forall}$

" for some " or " there is atleast one " or " there exists some "

note that " Some " is used in the sense of

" at least one " $\boxed{\exists}$

Write the Quantifiers of the following statements, where Predicate Symbols denote

$f(x)$: x is a fruit

$V(x)$: x is a vegatable.

$S(x,y)$: x is sweeter than y.

(a) Some vegetables are sweeter than all fruits.

$\exists x \; \forall(x) \wedge y \, [V(y) \to S(x,y)]$

(b) Every fruit is sweeter than all vegetables.

$\forall x \, (F(x) \wedge \forall y \, (V(y) \to S(x,y))$

(c) Every fruit is sweeter than some vegetables.

$\forall x \, (F(x) \wedge \exists y \, (V(y) \to S(x,y))$

(d) only fruits are sweeter than vegetables.

$\forall x \, \exists y, \; S(x,y) \to F(x)$

Free & Bound Variables

~~The~~

Free Variable: The variables which are not bounded by the Quantifiers are called free variables.

Example:- $P(x), Q(x)$: here x is a free variable.

Bounded Variables.

The variable which are bounded by Quantifiers are called bounded variables.

Example: $\forall(x) P(x), (\exists x) Q(x)$ here x is a bounded variable

Given a formula containing a part of the form $(x) P(x)$ or $(\exists x) P(x)$, such a part is called an x-bound part of the formula.

Any occurrence of x is an x-bound part of a formula is called a bound occurrence of x, while any occurrence of x or of any variable that is not a bound occurrence is called a free occurrence & (the formula $P(x)$ either in $(x) P(x)$ or in $(\exists x) P(x)$ is described as the scope of the quantifier. In other words, the scope of a Quantifier is the formula immediately following the Quantifier).

Consider the following formulas

1. $(x)\ P(x,y)$
It is the scope of the Quantifier & occurrence of x is bounded occurrence, while the occurrence of y is a free occurrence,

2. $(x)\ (P(x) \rightarrow Q(x))$
The scope of the universal Quantifier is $P(x) \rightarrow Q(x)$ & all occurrence of x are bound.

3. $(x)\ (P(x) \rightarrow (\exists y)\ R(x,y))$
The scope of (x) is $P(x) \rightarrow (\exists y)\ R(x,y)$. while the scope of $(\exists y)$ is $R(x,y)$. All occurrence of both x & y are bound occurrence.

4. $(\exists x)\ (P(x) \wedge Q(x))$. It is the scope of the Quantifier.

5. $(\exists x)\ (P(x) \vee Q(x))$
the scope of $(\exists x)$ is $P(x)$ and the less (double) occurrence of x is $Q(x)$ is free,

Note (i) In symbolizing Expression of the type "All A are B" the Correct Connective that should be used is the Conditional".

(ii) In symbolizing Expression of the type "Some A are B" the Correct Connective is the Conjunction

1. All Cats are animals.

$C(x)$: x is a Cat

$A(x)$: x is an animal,

$\forall x\,(C(x) \rightarrow A(x))$

2. Some Cats are black.

$\exists x\,[C(x) \land B(x)]$

3. All men are giants.

$G(x)$: x is a giant

$M(x)$: x is a man

$\forall x\,(M(x) \rightarrow G(x))$

4. All men are mortal.

For all x, if x is a man, <u>then</u> x is a mortal.

$M(x)$: x is man

$H(x)$: x is a mortal.

$\forall x\,(M(x) \rightarrow H(x))$

5. Any integer is either +ve or -ve

For all x, if x is an integer, then x is either +ve or -ve.

$I(x)$: x is an integer.

$P(x)$: x is either +ve or -ve.

$(\forall x)\,(I(x) \rightarrow P(x))$

6. There Exists a man

$(\exists x)(H(x))$ $H(x)$: x is a man.

7. Some men are clever.

$H(x)$: x is a man

$C(x)$: x is clever

$(\exists x)(H(x) \cap C(x))$

8. Some real numbers are rationals

$R_1(x)$: x is a real number.

$R_2(x)$: x is rational.

$(\exists x)(R_1(x) \cap R_2(x))$

There are two main Quantifiers - all & some.

The Quantifier "all" is called the universal Quantifier & denotes it by $\forall x$ [for all x, for every x, for each x]

The Quantifier "Some" is the existential Quantifier & denote it by $\exists x$ [there exists, there is at least one, Some]

Indicate the Variables that are ~~free~~ free & bound and write the scope of the Quantifiers:

(1) $(x)\,(P(x) \wedge R(x)) \rightarrow (x)\, P(x) \wedge Q(x)$

The scope of the first Quantifier (x) is $P(x) \wedge R(x)$, the scope of the Second Quantifier (x) is $P(x)$ and the last occurrence of x in $Q(x)$ is free while the first two occurrences of x are bound occurrence.

(2) $(x)\,(P(x) \wedge (\exists x)\, Q(x)) \vee ((x)\, P(x) \rightarrow Q(x))$

The scope of the first Quantifier (x) is $P(x) \wedge (\exists x)\, Q(x)$ while the ~~other~~ scope of $\exists(x)$ is $Q(x)$, the scope of the Second Quantifier (x) is $P(x)$ and the last occurrence of x in $Q(x)$ is free while the other occurrences of x are bound occurrences.

(3) $(x)(P(x) \rightarrow Q(x) \wedge (\exists x)\, R(x)) \wedge S(x)$.

The scope of the Quantifier (x) is $\wedge (\exists x)\, R(x)$ while the scope of the Quantifier $(\exists x)$ is $R(x)$ the last occurrence of x in $S(x)$ is free while the other occurrences of x are bound occurrences.