

**2/4 B.Tech. FOURTH SEMESTER****EE4T1 COMPLEX VARIABLES AND SPECIALFUNCTIONS Credits: 3****Lecture: 3 periods/week****Internal assessment: 30 marks****Tutorial: 1 period /week****Semester end examination: 70 marks****Course objective:**

After the completion of this course students will learn the concepts of analyticity of complex functions and find complex potential functions. They will be able to solve improper integrals, get familiar with conformal mappings. It enables them to know the solution of Bessel, Legendre equations and properties of Bessel function, Legendre polynomials.

**Course outcomes:** At the end of the course student will be able to

1. Determine complex potential function and summarize the properties of elementary functions of complex variables.
2. Solve the integration problems making use of Cauchy's integral formula.
3. Expand given function as power series using Taylor's, Laurent's series.
4. Determine the image of given region under the given conformal mapping
5. Comprehend Bessel functions, Legendre polynomials, related properties.

**UNIT I**

**Functions of a complex variables:** Continuity –Differentiability –Analyticity –Properties – Cauchy Riemann equations in Cartesian and polar coordinates. Harmonic and conjugate harmonic functions –Milne – Thompson method.

Elementary functions: Exponential, trigonometric, hyperbolic functions and their properties- General power  $z^c$  ( $c$  is complex), principal value.

**UNIT II**

**Complex Integration and series expansions:**

Line integral – Evaluation along a path and by indefinite integration – Cauchy's integral theorem – Cauchy's integral formula – Generalized integral formula.

Complex power series: Radius of convergence – Expansion in Taylor's series, Maclaurin's series and Laurent series. Singular points – Isolated singular point – pole of order  $m$  – essential singularity.

**UNIT III**

**Residues and evaluation of improper of integrals:** Evaluation of residues by formula and by Laurent series-Residue theorem. Evaluation of integrals of the type

a) improper real integrals  $\int_{-\infty}^{\infty} f(x) dx$       b)  $\int_0^{2\pi} f(\cos \theta, \sin \theta) d\theta$       c)  $\int_{-\infty}^{\infty} e^{imx} f(x) dx$

d) Evaluation of improper integrals by indentation.

**UNIT IV**

**Conformal mapping:** Transformation by  $e^z$ ,  $\ln z$ ,  $z^2$ ,  $z^n$  ( $n$  positive integer),  $\sin z$ ,  $\cos z$ ,  $z+a/z$ , translation, rotation, inversion and bilinear transformation-fixed point-cross ratio-properties-invariance of circles and cross ratio-determination of bilinear transformation mapping 3 given points.

**UNIT V**

**Special functions:** Bessel functions-properties-recurrence relations-orthogonality. Legendre polynomials-properties- Rodrigue's formula-Recurrence relations –orthogonality.

## Learning Resources

### Text Books:

1. Higher Engineering Mathematics – Khanna Publishers – B.S. Grewal – 42nd Edition: 2012, June.
2. Higher Engineering Mathematics – S.Chand Technical Publishers – H.K Dass.

### Reference Book:

Engineering Mathematics (Volume – III) - S. Chand - T. K. V. Iyengar, B. Krishna Gandhi, S. Ranganatham, M.V.S.S.N. Prasad- 9<sup>th</sup> Revised Edition: 2012